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Superstring Theory and Triple Systems

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1 Introduction

It has been expected that there exists M-theory, which unifies string theories. In M-theory, some structures of 3-algebras were found recently. First, it was found that by using $u(N)\oplus u(N)$ Hermitian 3-algebra, we can describe a low energy effective action of N coincident supermembranes [1–5], which are fundamental objects in M-theory.

Second, recent studies have indicated that there also exist structures of 3-algebras in the Green-Schwartz supermembrane action, which defines full perturbative dynamics of a supermembrane. It had not been clear whether the total supermembrane action including fermions has structures of 3-algebras, whereas the bosonic part of the action can be described by using a tri-linear bracket, called Nambu bracket [6, 7], which is a generalization of Poisson bracket. If we fix to a light-cone gauge, the total action can be described by using Poisson bracket, that is, only structures of Lie algebra are left in this gauge [8]. However, it was shown under an approximation that the total action can be described by Nambu bracket if we fix to a semi-light-cone gauge [9]. In this gauge, the eleven dimensional space-time of M-theory is manifest in the supermembrane action, whereas only ten dimensional part is manifest in the light-cone gauge. Moreover, 3-algebra models of M-theory itself were proposed and have been studied in [9–13].

The hermitian (ϵ,δ)-Freudenthal-Kantor triple systems [14–36] are generalizations of the hermitian 3-algebras [1–9, 37–71]. The hermitian 3-algebras are special cases, where $K(a,b)=0$ or equivalently, $<abc>=-<cba>$, of the hermitian $(-1,-1)$-Freudenthal-Kantor triple systems of second order. And the hermitian 3-algebras are classified into the $u(N)\oplus u(M)$ and $sp(2N)\oplus u(1)$ hermitian 3-algebras [13, 43, 45, 46, 52]. Therefore, it is natural to extend these triple systems to more general hermitian $(-1,-1)$-Freudenthal-Kantor triple systems or hermitian generalized Jordan triple systems.

In the following section, we summarize some results concerning with the generalization of the hermitian 3-algebras in M-theory [72, 73].

2 Definitions

Let us start with a definition of a *-(ϵ,δ)-Freudenthal-Kantor triple system.

Definition. A triple system U is said to be a *-(ϵ,δ)-Freudenthal-Kantor triple system if relations (0)–(iv) satisfy;

0) U is a Banach space,

i) $[L(a,b), L(c,d)] = L(< abc >, d) + \varepsilon L(c, < bad >)$,
ii) $K(< abc >, d) + K(c, < abd >) + \delta K(a, K(c,d)b) = 0$,
where $L(a,b)c =< abc >$ and $K(a,b)c =< acb > -\delta < bca >$, $\varepsilon = \pm 1, \delta = \pm 1$,
iii) $< xyz >$ is **C**-linear operator on x, z and **C**-anti-linear operator on y,
iv) $< abc >$ continuous with respect to a norm $|| \ ||$ that is, there exists $K > 0$ such that

$$|| < xxx > || \leq K||x||^3 \text{ for all} x \in U.$$

Furthermore, a *-(ϵ, δ)-Freudenthal-Kantor triple system is said to be hermitian if it satisfies the following condition,
v) all operator $L(x, y)$ is a positive hermitian operator with a hermitian metric

$$(x, y) = tr\ L(x, y),$$

that is, $(L(x,y)z, w) = (z, L^*(x,y)w)$, and $(x,y) = \overline{(y,x)}$.

Let U be a *-(ϵ, δ)-Freudenthal-Kantor triple system. Then we may define the notation of tripotent as follows.

Definition. It is said to be a tripotent of U if

$$< ccc >= c, \ \ c \in U.$$

3 Tripotent basis

In this section, we give decomposition theorems based on the tripotent basis.

Theorem 1.1. Let U be a hermitian $(-1, \delta)$-Freudenthal-Kantor triple system. If $W \subset U$ is flat (that is, $L(x,y) = L(y,x)$ for all $x, y \in W$), then we have a decomposition,

$$W = \mathbf{R}e_1 \oplus \cdots \oplus \mathbf{R}e_n$$

where e_i are tripotents or bitripotens.

Proof. See [72, 73].

We define the odd power of x inductively as follows;

$$x^{(3)} :=< xxx >,$$
$$x^{(2n+1)} :=< xx^{(2n-1)}x > .$$

By using this theorem, we have

Theorem 1.2. Let U be a hermitian $(-1, \delta)$-Freudenthal-Kantor triple system. Then every $x \in U$ can be written uniquely

$$x = \lambda_1 e_1 + \lambda_2 e_2 + \cdots + \lambda_n e_n,$$

where the e_i are tripotents or bitripotents, which are linear combinations of power of x, and the λ_i satisfy

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n.$$

Poof. See [72, 73].

4 Peirce decomposition

In this section, we give a theorem on a Peirce decomposition of a *-$(-1,-1)$-Freudenthal-Kantor triple system equipped with the tripotent $< ccc >= c$.

Theorem 2.1. Let U be a *-$(-1,-1)$-Freudenthal-Kantor triple system. Then, we have a decomposition with respect to a tripotent c (i.e.,$< ccc >= c$) as follows

$$U = U_1(c) \oplus U_{\frac{1}{2}}(c) \oplus U_0(c),$$

where

$$\begin{aligned}
U_1(c) &= \{x|(L(c,c)+R(c,c))x = 0,\ (R(c,c)-Id)x \neq 0\},\\
U_{\frac{1}{2}}(c) &= \{x|(L(c,c)+R(c,c))x \neq 0,\ (R(c,c)-Id)x = 0\},\\
U_0(c) &= \{x|(L(c,c)+R(c,c))x = 0,\ (R(c,c)-Id)x = 0\}.
\end{aligned}$$

Poof. See [72, 73].

5 Generalized hermitian 3-algebra

In this section, we extend the $u(N)\oplus u(M)$ 3-algebras to a hermitian $(-1,-1)$-Freudenthal-Kantor triple system.

Let $D^*_{N,M}$ be the set of all $N \times M$ matrices with operation

$$< xyz >= x\bar{y}^T z - z\bar{y}^T x + z x^T \bar{y},$$

where x^T and $\bar{x}$ mean transpose and conjugation of x, respectively.

Then $D^*_{N,M}$ is a hermitian $(-1,-1)$-Freudenthal-Kantor triple system. In fact, it satisfies the conditions (0), (i),(ii),(iii),(iv) and (v). This is an extension of the $u(N) \oplus u(M)$ hermitian 3-algebra, $< xyz >= x\bar{y}^T z - z\bar{y}^T x$, which is a basis for the effective action of the multiple membranes in M-theory.

One of the tripotents is given by

$$c = \begin{pmatrix} Id & 0 \\ 0 & 0 \end{pmatrix},$$

where Id is a $n \times n$ identity matrix ($n \leq N, M$). Because any element is decomposed as

$$x = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(A - A^T) & B \\ 0 & D \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(A + A^T) & 0 \\ C & 0 \end{pmatrix},$$

the Peirce decomposition is given by

$$\begin{aligned}
U_1(c) &= \{\begin{pmatrix} \frac{1}{2}(A - A^T) & B \\ 0 & D \end{pmatrix}\},\\
U_{\frac{1}{2}}(c) &= \{\begin{pmatrix} \frac{1}{2}(A + A^T) & 0 \\ C & 0 \end{pmatrix}\},\\
U_0(c) &= 0.
\end{aligned}$$

As in Theorem 1.1, we can expand any element as $x = \Sigma(\lambda_{ij}E_{ij} + \mu_{ij}\sqrt{-1}E_{ij})$, where E_{ij} means that (i,j) element is 1 and other element is zero, and E_{ij} and $\sqrt{-1}E_{ij}$ are tripotents, i.e., $< E_{ij}E_{ij}E_{ij} >= E_{ij}$, and $< \sqrt{-1}E_{ij}\sqrt{-1}E_{ij}\sqrt{-1}E_{ij} >= \sqrt{-1}E_{ij}$.

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